

The M-layer – overview of structure

Blair Hall

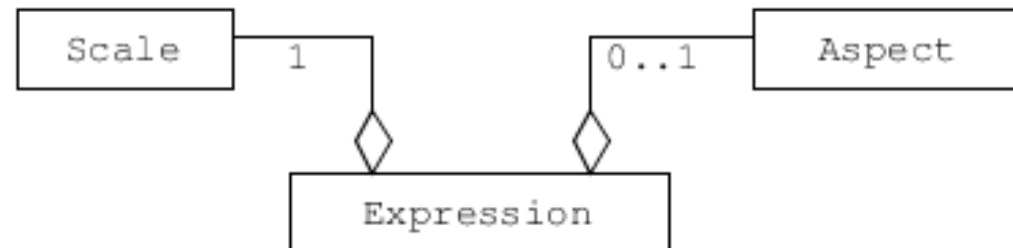
(not for circulation)

Expressions

(conventional) *expression* = $\{x\} [x]$
= 1.5 nm (e.g.)

(M-layer) *expression* = $\{x\} [[x]] \mid \{x\} [[x]] \langle x \rangle$
(e.g.) $\{1.5\} [[\text{nm}]] \text{ratio}$
(or) $\{1.5\} [[\text{nm}]] \text{ratio} \langle \text{length} \rangle$

$\{x\}$ – value
 $[[x]]$ – *scale* (combines a unit with a scale type)
 $\langle x \rangle$ – *aspect* (kind of quantity)



Aspect

Aspect is an optional third component in the expression of measured data. It may be thought of as the 'kind of quantity' but is more general in scope.

It's role is to disambiguate scale conversions (so it is not always needed)

An aspect identifies some property that can be measured, such as a physical quantity.

Aspect

```
{  
  "_entry_": "Aspect",  
  "uid": [  
    "ml_plane_angle",  
    95173225557230344956477808929590724690  
  ],  
  "locale": {  
    "default": {  
      "name": "plane-angle",  
      "symbol": "ang"  
    }  
  }  
},
```

$0.555 \text{ Hz} \longrightarrow 0.555 \text{ s}^{-1} \longleftarrow 0.555 \text{ Bq}$

$0.555 \text{ Hz} \xleftarrow{\text{frequency}} 0.555 \text{ s}^{-1} \xrightarrow{\text{activity}} 0.555 \text{ Bq}$

$1.5 \text{ N} \cdot \text{m} \longrightarrow 1.5 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \longleftarrow 1.5 \text{ J}$

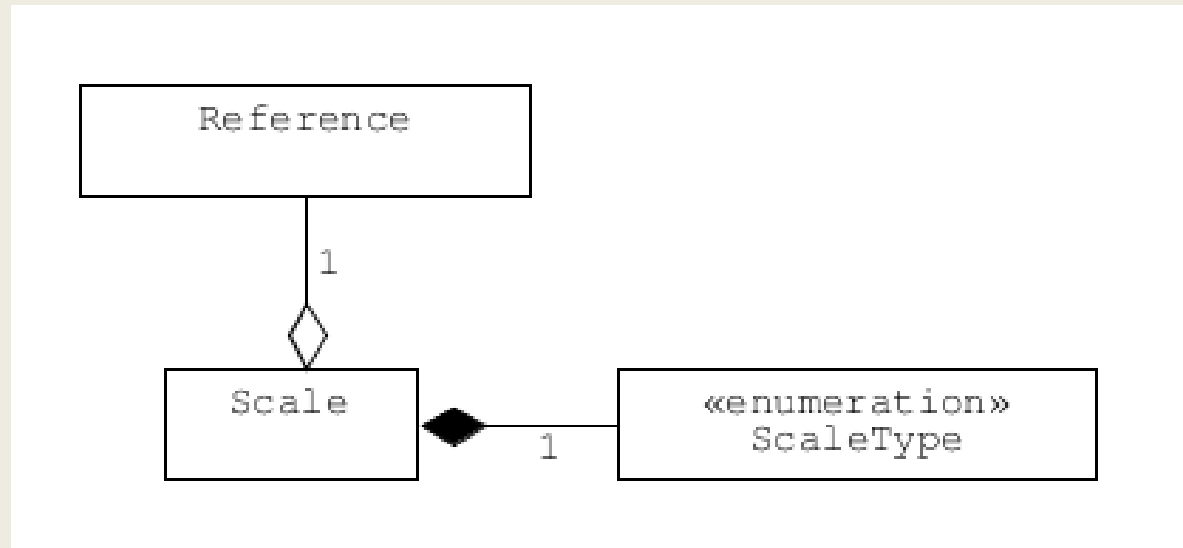
$1.5 \text{ N} \cdot \text{m} \xleftarrow{\text{torque}} 1.5 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \xrightarrow{\text{work}} 1.5 \text{ J}$

Scales

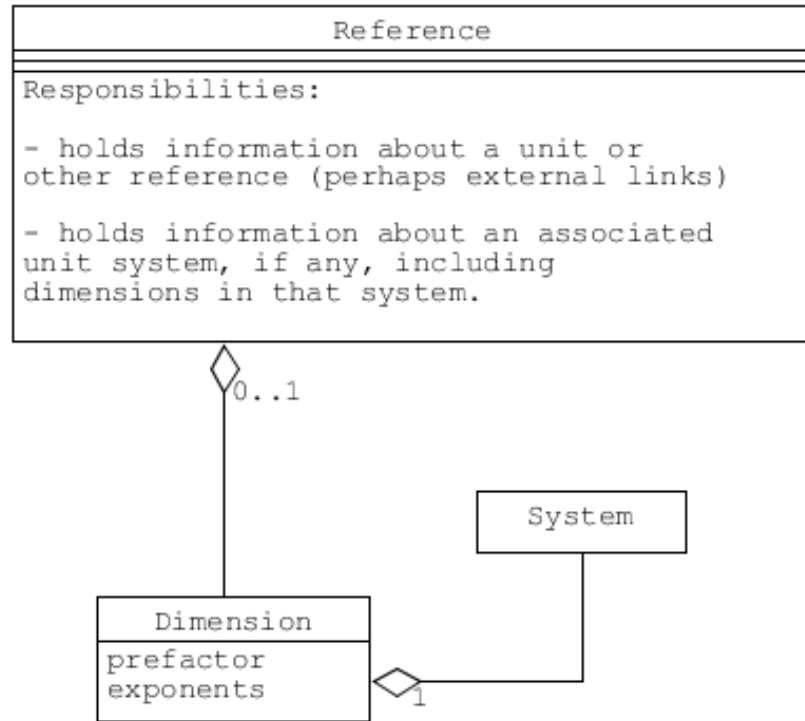
A scale associates a Reference with a particular type of scale.

We also designate some scales as “systematic” (when the name is composed of products of powers of base units or prefixed base units).

```
{  
  "__entry__": "Scale",  
  "uid": [  
    "ml_si_radian_ratio",  
    273301153578020696303516833405033923738  
  ],  
  "reference": [  
    "si_radian",  
    311561499536022904630229742672799278060  
  ],  
  "scale_type": "ratio"  
},
```



References – to existing standards



```
{
  "_entry_": "Reference",
  "uid": [
    "si_kilogram",
    188151331508313165897603768130808181784
  ],
  "locale": {
    "default": {
      "name": "kilogram",
      "symbol": "kg"
    }
  },
  "system": {
    "uid": [
      "si_system",
      88156805987886421108624908988601219537
    ],
    "dimensions": "[1, 0, 0, 0, 0, 0, 0]",
    "prefix": [
      "1",
      "1"
    ]
  },
  "UCUM": {
    "code": "kg",
    "description": "kilogram"
  }
},
```

Unit systems

The system contains a sequence of 'n' links to References, which are base units of the system. This establishes the order of dimensional exponents for units belonging to the system.



```
{
  "__entry__": "UnitSystem",
  "uid": [
    "si_system",
    88156805987886421108624908988601219537
  ],
  "name": "SI",
  "basis": [
    [
      "si_kilogram",
      188151331508313165897603768130808181784
    ],
    [
      "si_metre",
      61268972265076316018593147152102406832
    ],
    [
      "si_second",
      110730041758233939215703442037761569190
    ],
    [
      "si_ampere",
      264081801568151063132838497538090031099
    ],
    [
      "si_kelvin",
      25703533220788919988679332108037098600
    ],
    [
      "si_mole",
      96713855510406467826626480289106173630
    ],
    [
      "si_candela",
      107700549721211215242458620140782394628
    ]
  ]
}
```

Scale type

Scale types help with conversion rules

Two scales of the same type and for the same aspect are equivalent in terms of the information carried by data.

There is a strict hierarchy among types, allowing certain scale type promotions without loss of information.

«enumeration» ScaleType
ratio
interval
bounded_interval
ordinal
nominal

Type	Invariance transforms
ratio	$x' = a x, a > 0$
interval	$x' = a x + b, a > 0$
bounded-interval	$x' = (a x - x'_{\text{low}}) \bmod x'_{\text{range}} + x'_{\text{low}}$ ¹
ordinal	$x' = \text{any monotonic increasing function of } x$
nominal	$x' = \text{any 1-to-1 substitution for } x$

¹The range of target values is $[x'_{\text{low}}, x'_{\text{low}} + x'_{\text{range}}]$. The operator 'mod' divides the left argument by the right and returns the remainder with the same sign as the right argument.

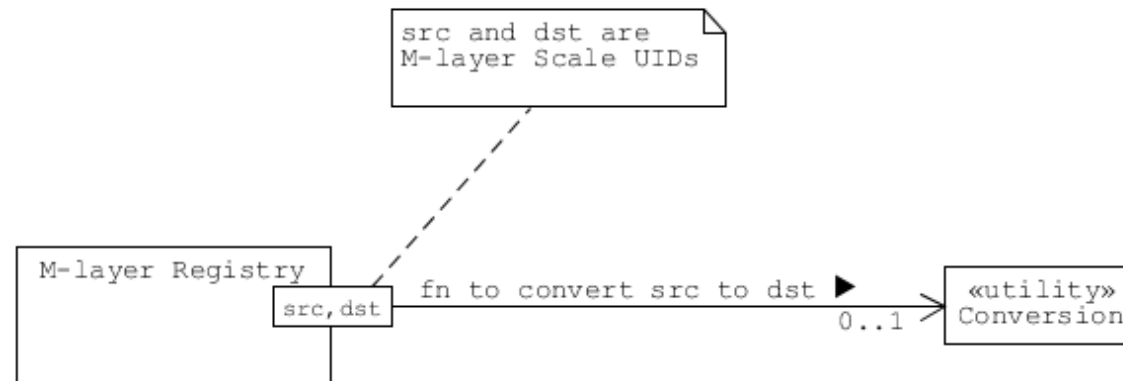
Generic conversion

Asymmetric rules are required

Unidirectional conversions are defined (without specifying an aspect).

```
{
  "__entry__": "Conversion",
  "src": [
    "ml_si_volt_ratio",
    324370471112617696659965827203196197232
  ],
  "dst": [
    "ml_si_kg.m2.s-3.A-1_ratio",
    99600280752733052556150747434649814921
  ],
  "function": "lambda x: x",
  "parameters": {}
},
```

scale-pair		function
src	dst	
[[m], ratio]	[[mm], ratio]	$y = x \times 10^3$
[[°C], interval]	[[kelvin], ratio]	$y = x + 273.15$

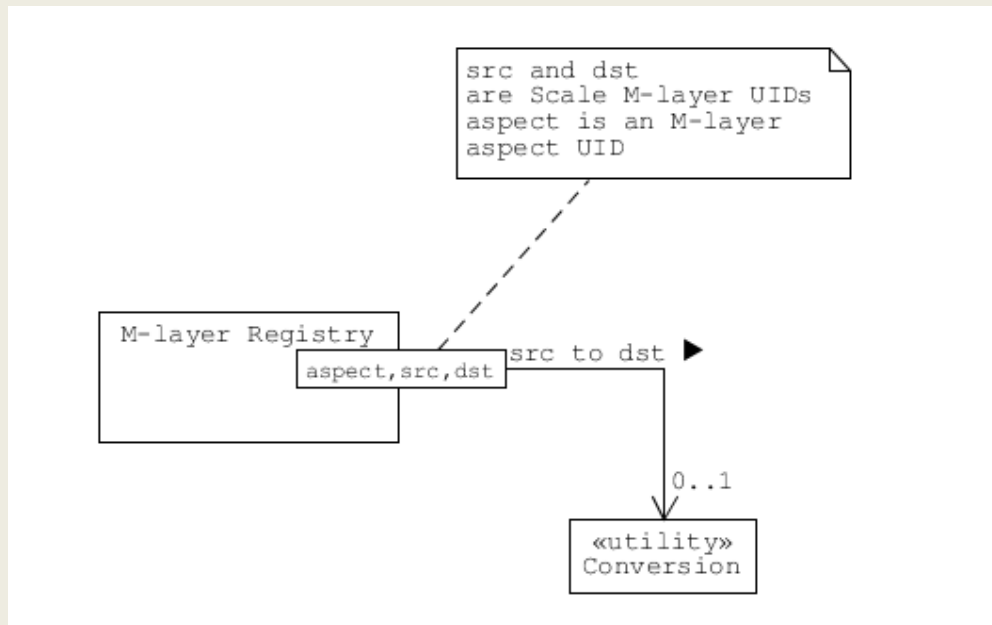


Aspect-specific conversion

Unidirectional conversions can be defined for specific aspects.

These definitions take precedence over generic conversion rules

aspect	scale-pair		function
	src	dst	
angle	[[rad], bounded]	[[°], bounded]	$y = (180/\pi x) \bmod 360$
temperature	[[kelvin], ratio]	[[°C], interval]	$y = x - 273.15$



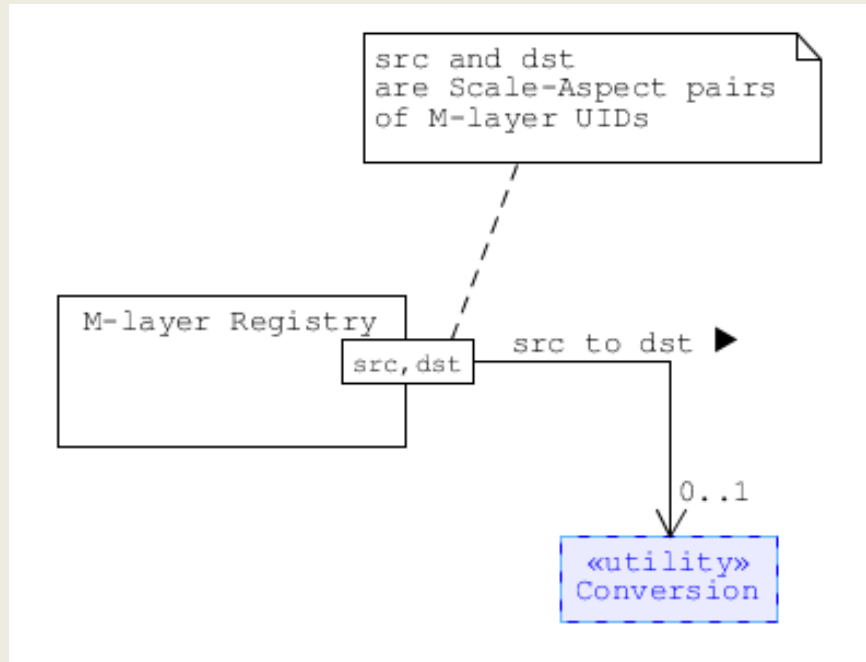
```
{
  "__entry__": "ScalesForAspect",
  "aspect": [
    "ml_energy",
    12139911566084412692636353460656684046
  ],
  "src": [
    "ml_si_kg.m2.s-2_ratio",
    92059200321763611267904598246242830863
  ],
  "dst": [
    "ml_si_joule_ratio",
    165050666678496469850612022016789737781
  ],
  "function": "lambda x: x",
  "parameters": {}
},
```

Casting

Castings are transformations that potentially change the information content of data.

To change an aspect requires casting and to change between scale types which may degrade information requires casting

scale-aspect src	scale-aspect dst	function
$[[\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}], \text{ratio}]$, $\langle \text{undefined} \rangle$	$[[\text{N} \cdot \text{m}], \text{ratio}]$, $\langle \text{torque} \rangle$	$y = x$
$[[\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}], \text{ratio}]$, $\langle \text{undefined} \rangle$	$[[\text{J}], \text{ratio}]$, $\langle \text{work} \rangle$	$y = x$



```
{
  "_entry_": "Cast",
  "src": [
    [
      "ml_si_s-1_ratio",
      323506565708733284157918472061580302494
    ],
    [
      "ml_no_aspect",
      295504637700214937127120941173285352815
    ]
  ],
  "dst": [
    [
      "ml_si_hertz_ratio",
      307647520921278207356294979342476646905
    ],
    [
      "ml_frequency",
      153247472008167864427404739264717558529
    ]
  ],
  "function": "lambda x: x",
  "parameters": {}
},
```



Measurement
Standards
Laboratory
of New Zealand



That's it 😊

A business of
CallaghanInnovation
New Zealand's Innovation Agency